Using the Algebra with Gaussians Spreadsheet

The new distribution formed by repeatedly combining random draws from two different Gaussian distributions, is itself a Gaussian. This is a very convenient property that is not true of other probability distributions.

We use the Greek letter Phi (a, b) notation to stand for a Gaussian distribution with **mean** = a and **variance** (standard deviation squared) = b.

The spreadsheet gives three different types of ways to add two Gaussians.

**First:** if the two Gaussians and re **Independent,** their correlation R, Covariance () and Mutual Information (are all 0. In that case, combining a draw from each results in a distribution with mean equal to the sum of the means, and variance equal to the sum of the variances. In Phi notation, we can write,

In the Spreadsheet, enter the mean [Cell C4] and variance [Cell E4] of the first Gaussian and the mean [Cell C6] and variance [Cell E6] of the second Gaussian.

The combined mean is given in [Cell C8] and the combined variance in [Cell E8].

**Second:** it is possible to create **weighted combinations** of the two Independent Gaussians. Most commonly, weights and are used so that + = 1, but any numbers can be used for w. In Phi notation,

In the Spreadsheet, enter the mean [Cell C13], variance [Cell E13], and weighting [Cell G13] of the first Gaussian and the mean [Cell C15], variance [Cell E15] and weighting [Cell G15] of the second Gaussian. The combined mean is given in [Cell C17] and the combined variance in [Cell E17].

Third: is it possible to create weighted (or unweighted) combinations of **dependent** Gaussian distributions. Dependence is expressed in terms of *Covariance*. In Phi Notation,

In the Spreadsheet, enter the mean [Cell C22], variance [Cell E22], and weighting [Cell G22] of the first Gaussian, the mean [Cell C24], variance [Cell E24] and weighting [Cell G24] of the second Gaussian, and the Covariance between them [Cell F22]. The combined mean is given in [Cell C26] and the combined variance in [Cell E26].

Example.

**Question.** I create an investment portfolio that is 65% Exxon stock and 35% Tesla Stock. The stocks’ expected annual returns have a Gaussian distribution, with Exxon = and Tesla = and the two distributions have Covariance = .03. What is the expected return and standard deviation of the investment portfolio?

**Answer.** Use the third spreadsheet section, for weighted combinations of dependent distributions. The expected return = 11.75% [Cell C26] and the expected standard deviation of return = 20.39% [Cell D26